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Moore and H. W. Tyler, was appointed for the purpose of coöperating with a similar committee, from the Society for the Promotion of Engineering Education, in an investigation of the teaching of mathematics in engineering schools.

(*g*) The Association received an invitation from the department of mathematics of Cornell University to hold its summer meeting at Ithaca in either of the years 1923 or 1924, in conjunction with the American Mathematical Society.

W. D. Cairns, *Secretary-Treasurer.*

MATHEMATICAL PROBLEMS IN THE WORK OF THE UNITED STATES COAST AND GEODETIC SURVEY.[1]

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In the office of the United States Coast and Geodetic Survey there are three divisions that employ mathematics to a considerable extent in the routine work which they perform. These are the Division of Geodesy, the Division of Tides and Currents, and the Division of Terrestrial Magnetism. We shall aim to give some account of the nature and extent of the mathematics required as a tool in each of these three divisions.

The work of the Division of Geodesy is almost entirely mathematical and the problems that are presented for consideration are many and varied. The routine work consists of the computation and adjustment of triangulation, the computation of astronomical observations, the computation of gravity observations, and the computation and adjustment of observations in precise leveling. This is a bill of fare that should be sufficiently comprehensive to suit the taste of any one who is mathematically inclined (or should we say, in accordance with popular belief, mathematically deranged?)

The computation of triangulation and of astronomical observations requires a thorough training in trigonometry with especial attention to the use of logarithmic tables. This training should be directed toward the attainment of both speed and accuracy, with a considerable premium placed on accuracy. It may be thought that common sense would suffice for the use of tables, and so it would seem; but, if so, good common sense is oftentimes sadly lacking in men when they graduate from college. In the examinations given by the Coast and Geodetic Survey, mistakes are made by candidates that are so utterly absurd that one can hardly explain the psychology of the errors.

The adjustment of observations by the method of least squares forms the greater part of the purely mathematical work in the general routine of the geodetic division. This adjustment finds its most extensive field of application when applied to triangulation. The regular method employed in the Survey for this purpose is that of adjustment with condition equations as distinguished from observation equations. The development of such condition equations for closures in length, azimuth, latitude, and longitude, requires considerable skill in the application of analysis to practical problems. These equations must be rigid enough to fulfil their purpose, but at the same time they must be as simple as possible so as to obviate the performance of any undue labor in the practical formation of the equations for any specific piece of triangulation.

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[1] Read at the meeting of the Ohio Section of the Mathematical Association of America, March 25, 1921.

The use of observation equations in the adjustment of triangulation, tried by other countries, has been found to have certain advantages. This Survey has worked out the necessary formulas and published them with examples for their use, but has not so far applied the method extensively to its own work. For the adjustment of differences of elevation, however, whether based on spirit leveling or trigonometric leveling, it is the practice of the Survey to use the method of observation equations, though it is quite practicable to use the method of condition equations. In the treatment of gravity observations and astronomical observations, observation equations are more frequently used than equations of condition.

The two largest adjustments that have ever been made in the Survey consisted of 264 and 284 equations. The amount of labor involved in the formation and solution of such a set of equations is enormous. In addition, one small error may require a month or two of hard work to make the necessary correction. It has been proposed to include all of the primary triangulation in the United States in one piece for scientific purposes, but since this would involve an adjustment consisting of between three and four thousand equations it has not as yet been attempted.

Whatever may be our opinion of the theory of least squares as founded upon probability, it must be recognized that it gives an exceedingly simple method for reconciling discrepancies in observations which, though small and practically insignificant, are yet troublesome because of the inconsistencies that they introduce. When one considers carefully, it is astonishing what degree of accuracy can be attained by careful work. The requirement for precise triangulation is that the average closing error of the triangles should be one second or less. Now on the largest circle used in horizontal angle measures (the 12 inch circle) one second represents about 58 one-millionths of an inch or about one and one half microns. With this fact in mind it is truly remarkable that such accuracy can be attained. By the adjustment these small outstanding discrepancies are reconciled and all values which have a check give one and the same result however computed.

Besides this regular routine work, the geodetic division prepares special publications on the theory of cartography, on triangulation, on the theory of gravity and isostatic compensation, and on various other phases of geodesy and geophysics. In this work practical applications may be found for many branches of mathematics. The whole subject of cartography comes finally under the head of the differential geometry of curves and surfaces. The subject of conformal mapping is founded directly upon the theory of the functions of a complex variable and forms the field for one of the most interesting practical applications of this branch of pure mathematics. The attempt to treat any of the projections in this class without consideration of its foundation upon the general theory of functions of a complex variable leads always to lack of clearness and definiteness of conception. Lagrange and Gauss have contributed much towards the generalization of the treatment of this kind of mapping. The equal-area mapping is a special case of equal-area representation of one surface upon another such as is treated in extended works on differential geometry. The perspective projections are practical applications of the principles of projective geometry. It is needless to say that trigonometry and calculus, both differential and integral, are employed in all of this cartographic work. Developments in Taylor series and especially those in Fourier series are frequently employed.

When tables are computed for any one of the projections, extensive logarithmic trigonometric computations have to be performed. In this work the judgment must be continually on the alert to determine the necessary exactness of the calculations required to attain the end desired. During the period of the war two extensive tables of this kind were prepared in the Division of Geodesy; the results are given in *Lambert Projection—Tables for the United States* (U. S. Coast and Geodetic Survey, Special Publication no. 52) and *Grid System for Progressive Maps in the United States* (U. S. Coast and Geodetic Survey, Special Publication no. 59). Both of these tables were prepared at the request of the army authorities and they were intended primarily for military purposes.

The tidal problem is still very far from a satisfactory solution despite the efforts of many of the ablest mathematicians since Newton's time. For purposes of prediction a modified form of the statical theory is used. This requires something of spherical harmonics and a great deal of development in series very similar to the developments needed in the lunar and planetary theories of dynamical astronomy, though not nearly so extensive. In questions of hydrodynamics involved in the tidal theory as so far developed, use is made (among other branches) of spherical harmonics, Fourier series, functions of a complex variable, Bessel functions, partial differential equations, and functions too recently introduced or too special to have any well-known name. Even the theory of numbers has been applied indirectly to tidal problems, for some of its theorems were found useful in deciding on the number of teeth to put on the gears of a tide-predicting machine.

The routine of computations regularly made in the Division of Tides and Currents calls for frequent use of the trigonometric functions in all four quadrants. Perhaps this is as good a place as any to call to mind what most teachers of trigonometry will sadly admit to be true, namely, that many students end their formal study of that subject with the idea that the signs of the trigonometric functions in the various quadrants and the process of looking up functions of an angle greater than 90° are great mysteries, so mysterious that even the rules for these processes are difficult to memorize.

In the Division of Terrestrial Magnetism the mathematician must be thoroughly familiar with his trigonometry and use of logarithms and must be prepared to use the principles of calculus and Fourier analysis in the reduction and discussion of the results. As the quantities measured are subject to continuous fluctuations, the results cannot be subjected to rigid adjustment as in the case of triangulation, but the method of least squares can frequently be used to advantage for interpolation or for separating errors of observation from fluctuations of which the character is established. Some of the fluctuations of the earth's magnetism are of a periodic character and the harmonic analysis may be used to deduce from the observations the periods and amplitudes of fluctuations which may be correlated with other terrestrial or cosmical phenomena. In the graphical representation of the distribution of the earth's magnetism over a given area it is sometimes desirable to determine the regular distribution corresponding most nearly to the observed results, and this may be done by the methods of least squares, using an analytical expression involving the first and second powers of the latitude and longitude.

While much of the work of the Division of Terrestrial Magnetism is of a routine character, involving little more than the application of arithmetic and logarithms to the use of formulas, yet even for this work the computer must understand the derivation of the formulas, know how to modify them when the character of the data justifies the use of approximations, and regulate the refinement of computation by the degrees of accuracy to be expected.

In the design of instruments for the Survey, various questions of mechanics and mathematical physics arise.

If then we consider not only the daily routine but also the theory underlying that routine and the necessity for improving methods and advancing knowledge, we shall not be surprised at the number and variety of the mathematical theories that are laid under contribution. But little more than an imperfect catalogue can be given here. Time and space would fail for any detailed treatment.

In Geodesy proper we have solid analytic geometry, differential geometry and elliptic functions in connection with measurement of the terrestrial spheroid. In connection with gravity problems, we need spherical harmonics, including zonal harmonics of the second kind, and in some cases curvilinear coördinates and ellipsoidal harmonics. In some problems integral equations are involved. In questions of geophysics intimately related to geodesy proper, such as the transmission of earthquake waves, integral equations are again involved. The transmission of earthquake waves requires, of course, the mathematical theory of elasticity as does also the mechanical problem of the variation of latitude. In questions of mapping, as has already been mentioned, differential geometry and the theory of functions of a complex variable and of trigonometric series are needed.

No matter how difficult or how advanced the theory on which a formula is based, the numerical application of that formula usually comes down to the use of tables and the four rules of arithmetic. In certain cases where arithmetical work on a large scale is to be done, it is possible to utilize the services of people who apply the rules of arithmetic to the data with little understanding of the reasons for what they are doing. Those who can do only this kind of unskilled labor—for it is mental labor corresponding to pick-and-shovel work in the field of manual labor—have been found to be unprofitable in the Coast and Geodetic Survey. They require too much supervision while at work and the supply of work that they can do is not continuous enough to ensure steady employment for them. For while there is much routine numerical work in the Coast and Geodetic Survey, there is a rather surprising amount of work that cannot be done by any cut-and-dried method. Peculiar conditions frequently arise in field work that must have special treatment in the reductions, and requests are continually coming in for information that the Survey would naturally supply, but which is not ground out by any of the regular machinery. Moreover, although a certain stability in form and method is desirable where great masses of data are handled, the results being published serially as they become available, nevertheless, no organization can keep on doing the same old thing in the same old way without suffering retrogression and decay. The Survey therefore seeks to learn and assimilate improved methods developed by other organizations doing similar work, to devise new methods on its own account and to advance knowledge in its own field.

Perhaps you will wonder, after hearing the foregoing list, not what branches of mathematics the Coast and Geodetic Survey needs, but what branches it does not need. You will find a few, however, that have not been mentioned. Perhaps the reason that some have been left out is that the right person has not come along to apply them; it may appear in the future that some branches of mathematics whose applicability to problems of mathematical physics is little known or not even suspected will turn out to be just what is needed to solve some problem in the work of the Coast and Geodetic Survey.

From their experience in the bureau, it seems to the authors desirable to lay stress on the following topics in elementary mathematics. These topics are important not alone in the work of the Coast and Geodetic Survey, but in other applications of mathematics to scientific and technical problems, and are not without cultural and disciplinary value. These concluding paragraphs are not to be taken as criticisms of present-day curricula and methods of teaching. The authors have both had some experience in teaching, and realize that, especially in pedagogical matters, it is easy to give advice, but much less easy to put it in practice.

1. *Numerical* calculation, first, with reference to *accuracy,* and second, *to economy of effort.* Very few young men just out of college or technical school have much idea of proportioning the accuracy of one part of a calculation to that of the other parts. They will cheerfully add together two quantities, one known to many decimal places, and the other known to much fewer, or they will multiply together two quantities, each known say to only three significant figures, and then retain the sixth significant figure of the product as if it meant something. It has never occurred to them that when the difference of two nearly equal quantities

is taken the number of accurate significant figures left is much reduced. These things are all simple enough, and are readily seen to be true once attention is drawn to them, but to many they seem to be considerations of a wholly new sort.

2. *Analytical trigonometry.* Somehow the signs of the trigonometric functions in the various quadrants should be taught so as to be retained, also the process of looking up functions of an angle greater than 90°, and the fact that the angle corresponding to a given function is not unique. Furthermore, a working knowledge of various trigonometric transformations is useful in applied mathematics.

3. *Approximations.* It is desirable to give students who are to apply their mathematics some instruction and training in the use of approximate formulas, in order that they may not distrust an admittedly approximate formula in those cases where it legitimately applies, and in order that, on the other hand, they may realize the limitations of such a formula and not try to make it do more than it is able to do.

AMONG MY AUTOGRAPHS.

By DAVID EUGENE SMITH, Columbia University.

12. The Marquise du Chastellet.

One of the letters in my collection is of special interest because of two lines. The letter is as follows:

> We have arrived in good health, Monsieur, but with disquietude which your own health causes us. I hoped that M. Cléraut would have had news from you. I beg of you to give me information about yourself, and to tell me if we shall see you soon. You do not realize the interest that I have. M. de Voltaire charges me to present to you his most affectionate compliments; and as for me, I am, with very profound friendship, Monsieur, your very humble and very obedient servant
>
> Breteuil du Chastellet.
>
> Paris, August 31, 1744.
>
> I hope that you will not forget our little affair.

So Voltaire sends by the Marquise du Chastellet (to adopt her own spelling of the name) his affectionate compliments to some one whom we do not know. She was entirely frank about delivering the message. Voltaire, who six years before this had essayed to make Newton intelligible to the French people, was with her. In 1735 they had begun their work together at the Chastellet country place at Cirey,—"that pitiless bore, the unfortunate Marquis," assenting, and conveniently spending some time with his regiment and in Paris, but accepting without protest the eighteenth-century customs even when at home. In July of the year 1744 they were again at Cirey, Voltaire, as usual, in bed and hard at work. He was fifty and the Marquise was thirty-eight. They went up to Paris the latter part of August, preparing for the festival of rejoicing at Louis XV's recovery from an illness and his return from the wars. There was also Voltaire's *Princess* that was soon to be produced, and this demanded his presence at the capital—a production that brought him much acclaim.